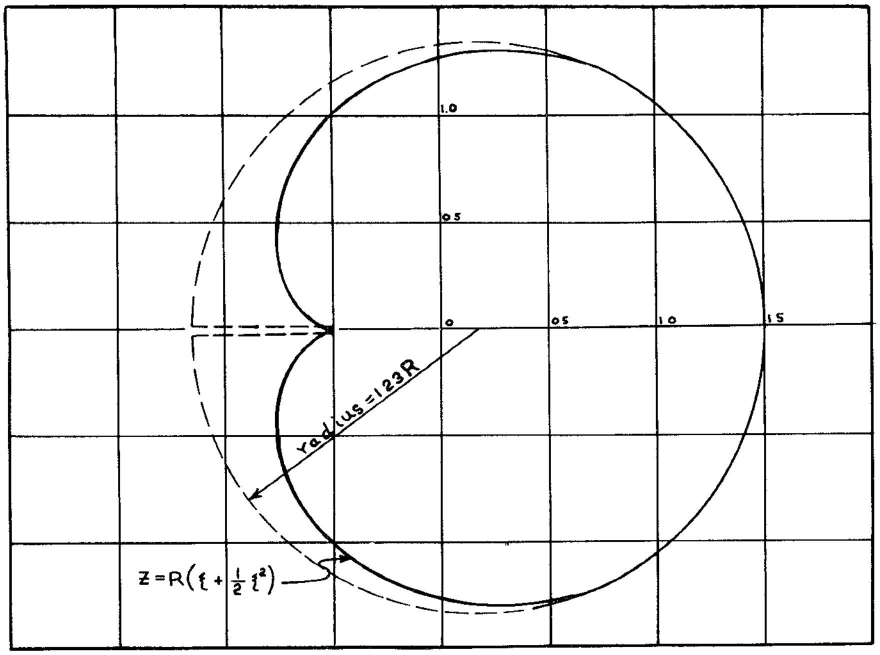


Fig. 1—Approximation of one-vaned waveguide by cardioid.

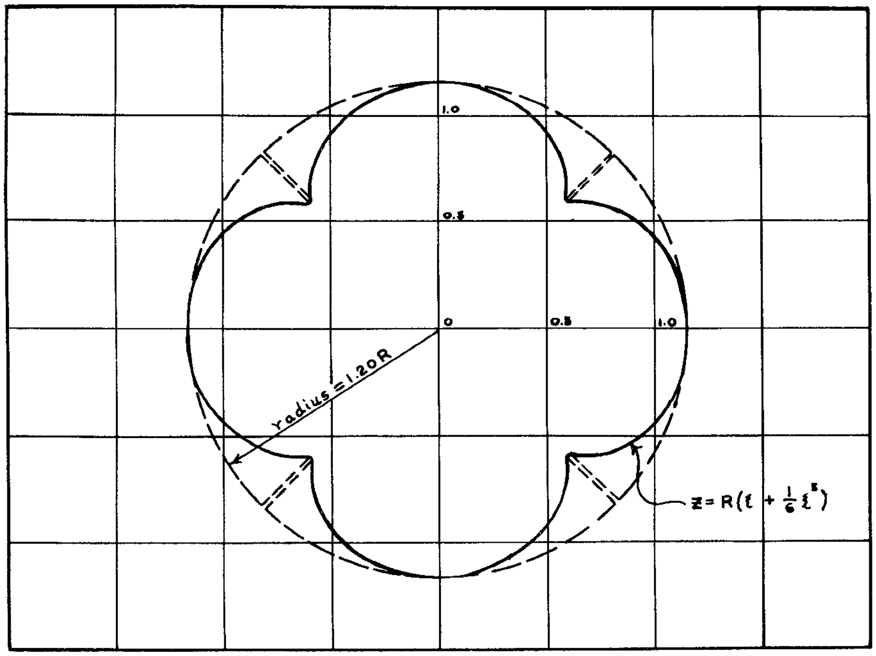


Fig. 2—Approximation of four-vaned waveguide by epitrochoid.

B. *Circular Waveguide with One Vane*

The transformation function for this configuration is difficult to determine. As a first approximation we represent the shape by a cardioid. The comparison of the wave guide and the mathematical model is shown in Fig. 1. The transformation function which maps a cardioid onto a unit circle is

$$z = R(\xi + \tfrac{1}{2}\xi^2).$$

By a procedure similar to that used in Case A we find

$$K_1 = \frac{2.163}{R}; \qquad K_2 = \frac{4.681}{R}.$$

Based on Case A, we can infer that the results may be a few per cent too high. It is interesting to note that the fundamental frequency of the circular guide without vanes (radius 1.23R) is about $1.96/R$. Comparing the corresponding results, we can conclude that the fundamental frequencies of a circular unvaned guide and vaned guide may differ as much as 10 per cent.

C. *Circular Waveguides with Four Diaphragms*

The shape of four-vaned circular guides may be represented by an epitrochoid (Fig. 2). The transformation function is

$$z = R(\xi + \tfrac{1}{6}\xi^5).$$

By a procedure similar to the tone used in Case A, we obtain the following frequency parameters:

$$K_1 = \frac{2.388}{R} \qquad K_2 = \frac{5.382}{R}.$$

In this case we have also used the bounding technique (2) to obtain a more accurate answer. After the necessary integration we obtain

$$\frac{2.384}{R} \leq K_1 \leq \frac{2.385}{R}.$$

The exact frequency parameter can be estimated with great certainty. A comparison can be made with an unvaned waveguide whose radius is about 1.2. The vaned waveguide whose cross section is representable by an epitrochoid has a fundamental frequency parameter about 20 per cent higher than that of the corresponding unvaned waveguide.

### Concluding Remarks

The method described in this correspondence appears to be suitable to estimate the frequency parameters of any arbitrarily shaped waveguide provided that the transformation function needed to map the section into a circular region can be obtained.

### Acknowledgment

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## Design Problems and Performance of Millimeter-Wave Fabry-Perot Reflector Plates

The parallel plate Fabry-Perot interferometer and its application have been described by several authors.[1-5] It is par-

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[1] W. Culshaw, "Reflectors for a microwave Fabry-Perot interferometer," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 221–228; April, 1959.

[2] ——, "High resolution millimeter wave Fabry-Perot interferometer," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 182–189; March, 1960.

[3] ——, "Resonators for millimeter and submillimeter wavelengths," IRE Trans. on Microwave Theory and Techniques, vol. MTT-9, pp. 135–144; March, 1961.

[4] ——, "Measurement of permittivity and dielectric loss with a millimetre wave Fabry Perot interferometer," Proc. IEEE, vol. 109, pt. B, Suppl. No. 23, pp. 820–826; 1961.

[5] R. W. Zimmerer, M. V. Anderson, G. L. Strine, Y. Beers, "Millimeter wavelength resonant structures," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 142–149; March, 1963.

ticularly useful as a resonator for which a very high $Q$ value and excellent transmission are required. The $Q$ values of parallel plate resonators can be described by the well-known expression

$$Q = m \frac{\pi}{2} \frac{1 + \Gamma^2}{1 - \Gamma^2}$$

where $m$ stands for the order of interference, given by the separation of the reflector plates, measured in units of half wavelengths. The parameter $\Gamma^2 = 1\text{-}a\text{-}t\text{-}d\text{-}r$ is determined by the various possible power losses as defined below.

- $a$ = power absorption coefficient of one reflector plate
- $t$ = power transmission coefficient of one reflector plate
- $d$ = power loss per single path due to diffraction
- $r$ = power loss per single path due to imperfections in flatness of the reflector plate which causes "off axis" reflections.

The increase of $Q$ values by working at a relatively high order of interference, is limited in the mm-wave region for practical reasons, since for $m > 100$ the diameter of the reflector plates becomes inconveniently large so that diffraction losses can be neglected. By choosing the transmission $t$ equal to the absorption of silver-plated perforated metal plates, maximum $Q/m$ values

$$\left(Q/m = \frac{\pi}{2} \frac{1 + \Gamma^2}{1 - \Gamma^2}\right)$$

in the order of 2000 can be predicted.

Provided diffraction losses can be neglected by working with sufficiently large plate diameters, deviations from this $Q/m$ value can be attributed to imperfections of the plate flatness, specified by the parameter $r$. With increasing plate separation, the flatness requirements become more severe.

It can be estimated that in the mm-wave region Fabry-Perot reflector plates should be flat within $1\mu$ to provide $Q/m$ values of approximately 2000 at a plate separation of 50 half wavelengths. For this plate separation, the plate diameter should be in the order of 200 mm[6] so that diffraction losses can be neglected, and thus it will be rather difficult to satisfy the flatness requirements.

This paper describes the results of various fabrication techniques used to make perforated metal plates[1] which meet the flatness requirement of mm-wave Fabry-Perot reflector plates and which are comparatively insensitive to mechanical and thermal influences. All measurements were performed at 70 Gc. The diameter of each plate was chosen to be 220 mm and the distance between the coupling holes and their diameter was 2.5 and 1.25 mm, respectively. With a plate thickness of 1 mm, the transmission of a single plate was measured to be 32 db. Except as otherwise stated, all $Q/m$ values were measured for $m = 50$. With perforated aluminum plates glued on supporting rings, $Q/m$ values could be achieved between 300 and 400.

[6] A. G. Fox and T. Li, "Resonant modes in a maser interferometer," *Bell Sys. Tech. J.*, vol. 40, pp. 435–488; March, 1961.

To show the influence of an insufficient flatness of the resonator plates, an attempt was made to stretch the plates in a manner similar to stretching the skin of a drum. One simple way to apply a homogeneous stretching is the use of thermal expansion forces. By heating the supporting rings electrically, $Q/m$ values could be increased to 800.

Instead of heating, we have also glued aluminum plates at temperatures of 50°C on supporting rings of stainless steel. Since steel has a smaller thermal expansion coefficient than aluminum, the plate will again be stretched if the system has cooled down to room temperature. Again $Q/m$ values in the order of 800 have been measured. These values are still smaller than those predicted theoretically. An investigation of the reflector plate flatness by means of an optical interferometer proved that the surface imperfections are still in the order of $10\mu$. Lapping and polishing methods were tried to correct the imperfections, but were unsuccessful because of an insufficient rigidity of the plates. To increase the rigidity of the plates, only the coupling area of a one-inch thick aluminum plate was machined down to a thickness of 1 mm. The whole plate consisted of the same material to avoid mechanical strains due to temperature changes.

The diameter of the coupling area was 125 mm and consisted of 2500 equally spaced coupling holes. After the plates had been electrolytically silver plated, each was lapped and polished while the flatness was consistently controlled with an optical interferometer. Thus, it was possible to achieve a flatness of $1\mu$ over the entire surface area of the reflector plate.

Measured $Q/m$ values of 1800 for these plates are close to those predicted theoretically. The diagram of Fig. 1 depicts the measured $Q$-values and the insertion loss for a Fabry-Perot resonator with the above-described reflector plates as a function of the plate separation and thus the order of interference. The highest $Q$ value of 160,000 was measured at the 110th order of interference. For larger reflector plate distances, energy losses due to diffraction effects will be so significant that the $Q$ value will decrease. The insertion loss increases with an increased plate separation; at $Q$ values of 100,000 the insertion loss was measured to be 13 db.

It should be mentioned, however, that because of the unfavorable thickness to diameter ratio of the coupling area, the lapping of the described reflector plates is a rather lengthy and cumbersome task. In this respect, metal evaporated quartz plates seem to have very promising features, since they can be lapped with much less effort to the required flatness limits.

Therefore, in our Fabry-Perot resonator, copper evaporated quartz plates having a diameter of 220 mm and flatness imperfections less than $1/10\mu$ were also checked. The thickness of the copper film was larger than 3 skin depths; the distance between the center of the photo-etched coupling holes was the same as that of the above described metal plates; the diameter of the holes, however, had to be decreased to 0.5 mm in order to achieve again a transmission of 32 db. Despite the improved flatness of the

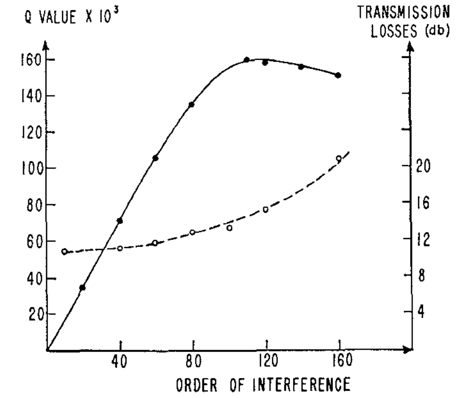


Fig. 1—$Q$ values and transmission losses of a Fabry-Perot resonator vs the order of interference.

quartz reflector plates, all $Q/m$ values have been measured to be slightly smaller than those of solid metal plates. These negative results can possibly be attributed to increased absorption losses caused by a slight deviation of the quartz plate thickness from the exact resonance condition as described by Zimmerer.[5]

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## A Differential Microwave Phase Shifter

It is well known[1,2] that a standard phase shifter can be constructed by combining a short-circuited section of uniform waveguide and a tuned, single directional coupler type of reflectometer, as shown in Fig. 1. In operation, the phase of the side arm output tracks the position of the short-circuiting plunger, once the tuning adjustments have been correctly made. The phase change $\phi$ corresponding to a displacement $l$ of the short circuit is

$$\phi = 2\beta l \qquad (1)$$

where $\beta = 2\pi/\lambda_G$ and $\lambda_G$ is the "guide wavelength."

The purpose of this note is to suggest a method of obtaining a differential phase shifter by combining two phase shifters of the above type with ganged short circuits, as shown in Fig. 2. The phase shifters are arranged so that the phase shift $\psi$ of the output is the difference of the phase shifts produced by the two units, or $\psi = \phi_2 - \phi_1$. If the uniform waveguide sections in which the short circuits slide have identical cross sections, then $\phi_1 = \phi_2$ and the phase shift, $\psi = 0$.

Manuscript received November 6, 1963.

[1] M. Magid, "Precision microwave phase shift measurements," IRE TRANS. ON INSTRUMENTATION, vol. I-7, pp. 321–331; December, 1958.

[2] G. E. Schafer and R. W. Beatty, "Error analysis of a standard microwave phase shifter," *J. Res. NBS*, vol. 64C, pp. 261–265; October–December, 1960.